On the correct model specification for estimating the structure of a currency basket

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A strand of literature has focused on inferring the weights of the anchor currencies in a country’s currency basket. In particular, with the growing importance of China’s economy, there has been a surging research interest in the role of the renminbi as an international anchor currency and in China’s de facto exchange rate regime. In this article, we show by a simple model and demonstrate using the observed data on the Special Drawing Rights (SDR) basket that the correct specification to estimate the weights of anchor currencies in a currency basket is to use the rates of change in exchange rates and to write exchange rates in quantity term. When one is to estimate the structure of a currency basket, close attention should be paid to using the rates of change or the levels of exchange rates as well as writing exchange rates in quantity or price term.

Keywords: Exchange rate regime, Currency basket, Currency weights, Currency amounts, renminbi.  
JEL classification: F31, F36
I. Introduction

There has been a continued interest in the distinction between the de facto and de jure exchange rate regimes. A strand of literature has focused on inferring the weights of the anchor currencies in a country’s currency basket. These works use a linear regression model and attempt to estimate the implicit weights of the anchor currencies. The weight-inference technique is popularized by Frankel and Wei (1994) and has been extensively used in the literature, for example, Kwan (1996), McKinnon and Schnabl (2004), and Bowman (2005). This technique has received tremendous attention recently. With the growing importance of China’s economy and the policy initiatives of the Chinese government to internationalize its currency, there has been a surging research interest in the role of the renminbi as an international anchor currency, for example, Chen et al. (2009), Ito (2010), Subramanian and Kessler (2013), and Fratzscher and Mehl (2014), and in China’s de facto exchange rate regime, for example, Ogawa and Sakane (2006), Yamazaki (2006), Frankel (2009), Fidrmuc (2010), and Fang et al. (2012). The majority of these studies employ the weight-inference technique.

To infer the weights in the currency baskets, Frankel and Wei (1994) propose a specification in the log differenced form:

$$
\Delta e_{re,t} = \alpha + \sum_{i=1}^{n} \beta_i \Delta e_{i,t} + \varepsilon_t
$$

(1)

where $\Delta$ stands for the difference, $e_{re}$ is the log of the exchange rate of the regional currency, $e_i$ is the log of the exchange rate of anchor currency $i$, all exchange rates are in price term (currency $i$ per unit of numeraire), $t$ stands for time and $\varepsilon_t$ for residuals. The estimated coefficient $\beta_i$ is inferred as the weight of anchor currency $i$ in the currency basket. On the other hand, Yamazaki (2006) and Fidrmuc (2010) propose the following specification in levels:
where the same notations apply and the estimated coefficient $\beta_i$ is inferred as the weight of anchor currency $i$.

Are both of these two specifications correct in the sense of allowing one to infer the weights of the anchor currencies using the estimated coefficients? Moreover, are the exchange rates to be expressed in quantity term (numeraire per unit of currency) or price term (currency per unit of numeraire)? This article aims at answering these two questions. We show that to estimate the weights of the anchor currencies in the basket, the correct specification is to use the rates of change in exchange rates and to write exchange rates in quantity term. If the estimation equation is based on the levels of exchange rates and exchange rates are in quantity term, the estimated coefficients are the amounts, not the weights, of the anchor currencies. When the estimation equation is based on the levels of exchange rates and exchange rates are in price term, the estimated coefficients are anything but the weights or amounts of the anchor currencies.

The rest of this article is as follows. Section II derives a simple model to estimate the structure of a currency basket and discusses the implications of different model specifications. Section III uses the observed data on the SDR basket to verify the implications of different model specifications. Section IV concludes this article.

II. The Model

For simplicity, suppose the regional currency (RC) follows a basket peg and the value of one regional currency is pegged to the value of a basket of currencies$^1$ which has $q_1$ unit of US dollar and $q_2$ unit of euro ($q_1 > 0$, $q_2 > 0$), then we have:

\[
E_{rc,t} = \alpha + \sum_{i=1}^{n} \beta_i E_{i,t} + \epsilon_t
\]

$^1$ The exchange rate of the regional currency (RC) and the basket of currencies can be written as $1 \text{ RC} = X \text{ Basket}$. As the size of the basket can be rescaled proportionally, it is more convenient to express the exchange rate as $1 \text{ RC} = 1 \text{ Basket}$. 

Value of 1 RC = Value of 1 basket of currencies

(3)

Values of the currencies in Equation 3 are measured by a common numeraire. Using the Swiss franc as the numeraire, Equation 3 can be rewritten as:

\[ E_{rc} = q_1E_{usd} + q_2E_{eur} \]  

(4)

where \( E_{rc} \), \( E_{usd} \), and \( E_{eur} \) are the exchange rates of the regional currency, USD, and EUR against CHF, expressed in quantity term (CHF per unit of currency). The left-hand side of Equation 4 is the CHF equivalent of one regional currency and the right-hand side is the CHF equivalent of one currency basket.

It should be noted that the coefficient \( q_i \) in Equation 4 is the amount of currency \( i \) in the basket, not the weight of currency \( i \). The weight of currency \( i \) \( (w_i) \) is:

\[ w_i = \frac{q_iE_i}{q_1E_{usd} + q_2E_{eur}} = \frac{q_iE_i}{E_{rc}} \]  

(5)

Written in differenced form and divided by \( E_{rc} \), Equation 4 becomes:

\[ \frac{\Delta E_{rc}}{E_{rc}} = q_1 \frac{\Delta E_{usd}}{E_{rc}} + q_2 \frac{\Delta E_{eur}}{E_{rc}} \]  

(6)

Rewrite \( q_i \frac{\Delta E_i}{E_{rc}} \) on the right-hand side of Equation 6 as \( \frac{q_iE_i}{E_{rc}} \frac{\Delta E_i}{E_i} \) and let \( \frac{q_iE_i}{E_{rc}} = w_i \) (recall that \( \frac{q_iE_i}{E_{rc}} \) is the weight of currency \( i \) by Equation 4), we have:

\[ \frac{\Delta E_{rc}}{E_{rc}} = w_1 \frac{\Delta E_{usd}}{E_{usd}} + w_2 \frac{\Delta E_{eur}}{E_{eur}} \]  

(7)

where \( w_1 \) and \( w_2 \) are the weights of the USD and EUR. Equation 7 can be interpreted
intuitively. If the USD, with a weight of $w_1$, appreciates by 1% against the CHF and other things being equal, the currency basket will appreciate by $w_1\%$ against the CHF. As the value of the regional currency is pegged to the value of the basket, the Monetary Authority must appreciate the regional currency by $w_1\%$ against the CHF to keep its exchange rate against the basket constant.

Three important points can be made on the model specifications for estimating the structure of a currency basket. First, to estimate the weights of the anchor currencies, the rates of changes in exchange rates should be used and exchange rates should be written in quantity term. For example, to infer the weights of the USD and EUR, one should estimate the following equation:

$$\frac{\Delta E_{rc,t}}{E_{rc,t}} = \alpha + w_1 \frac{\Delta E_{usd,t}}{E_{usd,t}} + w_2 \frac{\Delta E_{eur,t}}{E_{eur,t}} + \epsilon_t$$  \hfill (8)

where exchange rates are in quantity term, \(t\) stands for time and \(\epsilon_t\) for residuals. The estimated \(w_1\) and \(w_2\) are the weights of the USD and EUR, respectively. When the rates of changes in exchange rates are not large, Equation 8 can be approximated by the log differenced form:

$$\Delta e_{rc,t} = \alpha + w_1 \Delta e_{usd,t} + w_2 \Delta e_{eur,t} + \epsilon_t$$  \hfill (9)

where \(\Delta\) stands for the difference and \(\epsilon_t\) is the log of the exchange rate \(E_{ij}\).\(^2\)

Second, the estimated coefficients are the amounts of the anchor currencies if we regress the level of exchange rate of the regional currency on the levels of exchange rates of the anchor currencies and exchange rates are in quantity term:

$$E_{rc,t} = \alpha + \beta_1 E_{usd,t} + \beta_2 E_{eur,t} + \epsilon_t$$  \hfill (10)

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\(^2\) When Equation 9 is estimated, the estimated \(w_i\) will be the same regardless of the fact that exchange rates are in quantity term or price term. The specification proposed by Frankel and Wei (1994) is similar to Equation 9.
By Equation 4, we know that the estimated $\beta_1$ and $\beta_2$ in Equation 10 are the amounts of the USD and EUR in the basket. Therefore, it is incorrect to infer the estimated coefficients as the currency weights.

Third, if we regress the level of exchange rate of the regional currency on the levels of exchange rates of the anchor currencies and exchange rates are in price term, the estimated coefficients are anything but the amounts or weights of the anchor currencies. As exchange rate in price term is simply the inverse of exchange rate in quantity term, the regression equation in this case is:

$$(1/ E_{r,t}) = \alpha + \beta_1 (1/ E_{usd,t}) + \beta_2 (1/ E_{eur,t}) + \epsilon_t$$

By Equations 4 and 7, the estimated $\beta_1$ and $\beta_2$ of Equation 11 are neither the amounts nor the weights of the anchor currencies. We cannot infer the estimated coefficients as the currency weights. It will be cumbersome if not impossible to transform these coefficients to the currency weights. Examples that used this specification are Yamazaki (2006), Moosa et al. (2009) and Fidrmuc (2010).

The estimation of currency weights in the currency basket is analogous to the estimation of the weights of assets in a portfolio. Suppose we have a portfolio consisting of 1 share of IBM stock and 2 shares of Citibank stock and the prices of the IBM and Citibank stocks are $P_i$ (USD per share of IBM stock) and $P_c$ (USD per share of Citibank stock), then the value of this portfolio $V$ (USD per portfolio) is:

$$V = 2 \times P_i + 1 \times P_c$$

If we regress the rate of changes in $V$ on the rates of changes in $P_i$ and $P_c$, the estimated coefficients are the average weights of IMB and Citibank stocks in this portfolio for the estimation period. If we regress $V$ on $P_i$ and $P_c$, the estimated coefficients are the numbers of share of IMB and Citibank stocks in the portfolio. If we regress $1/V$ on $1/P_i$ and $1/P_c$, the estimated coefficients are anything but the weights or the numbers of shares of IMB and Citibank stocks.
III. Empirical Illustration using Data on the SDR Basket

This section uses the observed data of the Special Drawing Rights (SDR) basket to test for the implications of different model specifications derived in Section II. The IMF has announced that on January 1, 2011 the initial weights assigned to USD, EUR, GPB and JPY in the SDR basket are 41.9%, 37.4%, 11.3% and 9.3%. Effective from January 1, 2011, the currency amounts of the USD, EUR, GPB and JPY in the SDR basket are 0.660, 0.423, 0.111 and 12.1, which will be in effect to the end of 2015 (confer IMF, Currency amounts in new Special Drawing Rights basket).

Suppose we only know that the USD, EUR, GBP and JPY are in the SDR basket, how can we use the observed data to estimate the weights of these four currencies? We use three different model specifications discussed in Section II and daily data from January 1, 2011 to April 30, 2014, with Swiss franc as the numeraire, to do the task. Estimation results are reported in Table 1. All estimated coefficients in Table 1 are statistically significant at the 1% level.

In the first specification, exchange rates are in quantity term and the rate of changes in SDR exchange rate is regressed on the rates of changes in the exchange rates of the four anchor currencies. This is the correct specification shown in Section II. The estimated coefficients of the USD, EUR, GBP and JPY are 0.437, 0.362, 0.136 and 0.073, respectively, which are very close to the initial weights of these four currencies. And we can use these estimated coefficients to infer the weights of the USD, EUR, GBP and JPY in the SDR basket.

In the second specification, exchange rates are in quantity term and the level of exchange rate of SDR is regressed on the levels of exchange rates of the four currencies. We have shown in Section II that the estimated coefficients from this specification are the currency amounts. Using this specification, the estimated coefficients of the USD, EUR, GBP and JPY are 0.660, 0.423, 0.111 and 12.108, respectively, which are virtually identical to the actual amounts of these four currencies. If we use these coefficients to infer the currency weights, we will erroneously conclude that the JPY is the most important currency in the SDR basket.
and it is far more important than the other three currencies.

The third specification writes exchange rate in price term and regresses the level of exchange rate of SDR on the levels of exchange rates of the four currencies. As shown in Section II, the estimated coefficients from this specification are anything but the currency weights or amounts. Using this specification, the estimated coefficients of the USD, EUR, GBP and JPY are 0.273, 0.310, 0.136 and 0.001, respectively, which are indeed very different from the initial weights and amounts of these four currencies. If we use these coefficients to infer the currency weights, we will erroneously conclude that the EUR is the most important currency and the JPY is negligible in the SDR basket.

In sum, estimation results on the structure of the SDR basket are consistent with the predictions of our model in Section II and support the argument that the correct specification to estimate the weights of the anchor currencies in a currency basket is to use the rates of changes in exchange rates and to write exchange rates in quantity term.

IV. Conclusion

In this article, we have shown by a simple model and demonstrated using the observed data on the SDR basket that the correct specification to estimate the weights of the anchor currencies in a currency basket is to use the rates of changes in exchange rates and to write exchange rates in quantity term. When the estimation equation is based on the levels of exchange rates and exchange rates are in quantity term, the estimated coefficients are the amounts, not the weights, of the anchor currencies. If the estimation equation is based on the levels of exchange rates and exchange rates are in price term, the estimated coefficients are anything but the weights or amounts of the anchor currencies. Based on these findings, we suggest that close attention should be paid to using the rates of changes or the levels of exchange rates as well as writing exchange rates in quantity or price term when one is to estimate the structure of a currency basket.
Table 1. Estimation of the SDR basket using three different model specifications

<table>
<thead>
<tr>
<th>USD</th>
<th>EUR</th>
<th>GBP</th>
<th>JPY</th>
<th>Adj. $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Panel 1. rates of change in exchange rates are used and exchange rates are in quantity term</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.437</td>
<td>0.362</td>
<td>0.136</td>
<td>0.073</td>
<td>0.994</td>
</tr>
<tr>
<td>(104.25)</td>
<td>(90.99)</td>
<td>(31.06)</td>
<td>(25.76)</td>
<td></td>
</tr>
<tr>
<td>Panel 2. levels of exchange rates are used and exchange rates are in quantity term</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.660</td>
<td>0.423</td>
<td>0.111</td>
<td>12.108</td>
<td>0.999</td>
</tr>
<tr>
<td>(898.49)</td>
<td>(972.25)</td>
<td>(198.87)</td>
<td>(800.49)</td>
<td></td>
</tr>
<tr>
<td>Panel 3. levels of exchange rates are used and exchange rates are in price term</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.273</td>
<td>0.310</td>
<td>0.136</td>
<td>0.001</td>
<td>0.999</td>
</tr>
<tr>
<td>(376.52)</td>
<td>(407.38)</td>
<td>(96.30)</td>
<td>(348.84)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: All exchange rates use Swiss Franc (CHF) as the numeraire. Sample period is from January 1, 2011 to April 30, 2014 (868 observations). Figures in parentheses are t-statistics. The IMF has announced that on January 1, 2011 changes in the relative weights of the four currencies in the SDR basket come into effect. The initial weights assigned to USD, EUR, GPB and JPY are 41.9%, 37.4%, 11.3% and 9.3%. Effective from January 1, 2011, the currency amounts of USD, EUR, GPB and JPY are 0.660, 0.423, 0.111 and 12.1, which will be in effect to the end of 2015.
References
